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LETTER TO THE EDITOR

Mixing heat-bath and Glauber dynamics: damage spreading in the Ising model

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Abstract. We study the damage spreading in the Ising model, on a square lattice, by a Monte Carlo approach, using a general 'f-dynamics' ($0 \leqslant f \leqslant 1$) which reproduces, for $f=0$ the well known heat-bath dyanmics and for $f=1$ Glauber dynamics. We find, for initially small damage, a critical value $f_c = 0.50 \pm 0.03$ as the threshold for damage spreading. For $f_c < f \leqslant 1$ the damage is non-zero only for $T_c < T < T_0(f)$, where T_c is the usual critical temperature and $T_0(f)$ rises with f like $T_0(f) - T_c \propto (f-f_c)^\alpha$ ($\alpha = 0.83 \pm 0.08$). For $T_c \leqslant T$ the damage $D(T,f)$ seems to satisfy the scaling relation $D(T,f) \sim |\ln(f-f_c)|^{-1} \mathscr{F}((1-T_c/T)/(f-f_c)^\alpha)$.

One of the most puzzling aspects of the dynamics of statistical models, is the so-called damage spreading. It consists in studying the time evolution of two configurations of the model, submitted to the same microscopic dynamics, measuring the Hamming distance (or damage) between the two configurations (≡fraction of sites that are in different states in the two configurations).

In general, when the temperature goes from zero to infinity, we observe different behaviours: regions where the damage is hindered (frozen phases) and others where it spreads (chaotic phases). In recent years, several investions [1-18] have been made, for the particular case of the Ising model, in order to understand the relevant features of the frozen-chaotic phase transition. It is known that the details of the microscopic dynamics used to obtain the time evolution of the configurations plays a major role for the phenomenon. For Glauber and Metropolis dynamics, usually, the chaotic phase is at high temperatures [3, 4, 6-8], while in the heat-bath dynamics [1, 2, 5], this phase (if it exists) is at low temperatures.

Glauber and heat-bath dynamics, which in terms of probabilities give identical results for the evolution of a single configuration, have been found to behave differently as far as damage spreading (i.e. the compared evolution of two configurations) is concerned (compare [1] and [3]). To investigate this dependence of the damage spreading on the microscopic dynamics we study, in this letter, the zero-field ferromagnetic Ising model on a square lattice, using a general 'f-dynamics' ($0 \leqslant f \leqslant 1$) that continuously interpolates between heat-bath ($f=0$) and Glauber ($f=1$) dynamics.

To define this dynamics, we consider, at a time t, the local field $h_i(t)$ acting on site i ($S_i = \pm 1$) and an associated probaiblity $p_i(t)$, given by

$$p_i(t) = (1 + e^{-2h_i(t)})^{-1} \tag{1}$$

with

$$h_i(t) = \frac{J}{k_B T} \sum_j S_j(t) \tag{2}$$

where the j are the nearest-neighbouring sites of site i.

To determine the new value of S_i, at a time $t+1$, we select a random number $0 \leq \pi_i(t) \leq 1$, and apply the following rule:

$$S_i(t+1) = \begin{cases} 1 & \text{if } \pi_i(t) \leq p_i'(t) \\ -1 & \text{if } p_i'(t) < \pi_i(t) \leq p_i'(t) + [1 - p_i(t)] \\ 1 & \text{if } p_i'(t) + [1 - p_i(t)] < \pi_i(t) \end{cases} \tag{3}$$

with

$$p_i'(t) = \left(1 - \frac{f}{2}(1 + S_i(t))\right) p_i(t). \tag{4}$$

We see that the interval [0, 1] is divided in three pieces; the central one having a length $1 - p_i(t)$ and the others together a length $p_i(t)$. Depending on whether the random number falls in the central piece or any of the other ones, the new value of S_i is set to be -1 or $+1$ respectively. If $f = 0$ the third piece has zero length and we recover the heat-bath dyanamics. If $f = 1$ the first or the third piece have zero length depending on whether $S_i(t)$ have values $+1$ or -1 respectively, and the f-dynamics reproduces Glauber.

To study the damage spreading, we create, at time $t = 0$, two configurations of the model $\{S_i^A\}$ and $\{S_i^B\}$, and let them evolve following the above dynamics, using at each time, for both configurations, the same random number. The Hamming distance, or damage, is given by

$$D(t) = \frac{1}{2N} \sum_i |S_i^A(t) - S_i^B(t)| \tag{5}$$

where N is the total number of sites.

We have calculated $D(t)$ numerically, on a 40×40 square lattice. First we thermalise the configuration $\{S_i^A\}$ over 800 times steps per spin and, at the time $t = 0$, we create the configuration $\{S_i^B\}$ as $S_i^B = S_i^A$, with the exception of the central site, where $S_0^B = -S_0^A$. Therefore the initial damage is $D(0) = 1/N$. Next we let the damage evolve during a relaxation time of 800 time steps per spin and, after this, we take the time average of the damage over 1600 time steps per spin. We repeat these operations, using different sequences of random numbers for different samples until we have at least 80 samples where the damage is different from zero. Finally, we take the average over those samples to obtain the final damage.

We have observed the following.

(i) There is a threshold value of f ($f_c = 0.50 \pm 0.03$) for damage spreading.

(ii) For $f_c < f < 1$, the damage is non-zero only for $T_c < T < T_0(f)$, i.e. when the temperature T goes from zero to infinity, the model presents three phases: frozen ($0 \leq T \leq T_c$), chaotic ($T_c < T < T_0(f)$) and frozen ($T_0(f) \leq T$) (see figure 1).

(iii) Near T_c and $T_0(f)$ strong fluctuations appear in particular for f close to f_c, that make a precise measurement of the damage difficult. Nevertheless, we have been able to obtain an expression for $T_0(f)$ (within of the limits of our statistical error bars)

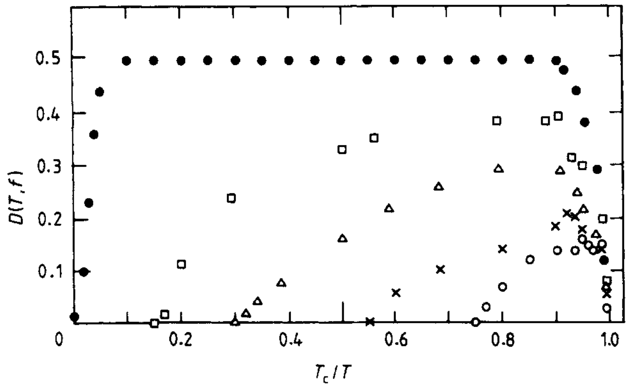

Figure 1. Average damage $D(T,f)$ against T_c/T (●, □, △, ×, ○ mean respectively $f = 1$, 0.9, 0.8, 0.7, 0.6).

that is given by $1 - T_c/T_0(f) = A(f - f_c)^\alpha$, where $A = 1.8 \pm 0.15$ and $\alpha = 0.83 \pm 0.08$ (see figure 2).

(iv) We tried to obtain a scaling law for the damage near T_c and $T_0(f)$. Near T_c the best scaling form that we found was $D(T,f) \sim |\ln(f - f_c)|^{-1}\mathscr{F}(X)$, where $X = (1 - T_c/T)/(f - f_c)^\alpha$ and $\mathscr{F}(X)$ is a scaling function (figure 2(*a*)). The best power-law scaling, given by $D(T,f) \sim (f - f_c)^\beta \mathscr{F}_1(X)$ ($\beta = 0.76 \pm 0.08$) does not show a data collapse as good as the one found with a logarithm (see figure 2(*b*)). Near $T_0(f)$ the damage is of the same order as the statistical error close to f_c and therefore we were not able to obtain a satisfactory scaling law.

These results show that the transition from heat-bath dynamics to Glauber dynamics is not simple; in fact, at the finite value $f_c \simeq 0.5$ one has a critical point between a Glauber-like behaviour in which a chaotic phase exists and a heat-bath-like behaviour without such a phase. Around the critical point f_c we found scaling laws.

It should be interesting to study, with more numerical precision, the damage near $T_0(f)$, in order to obtain the critical behaviour associated with this dynamical phase transition, and also to study this model in three dimensions and in the presence of an external magnetic field.

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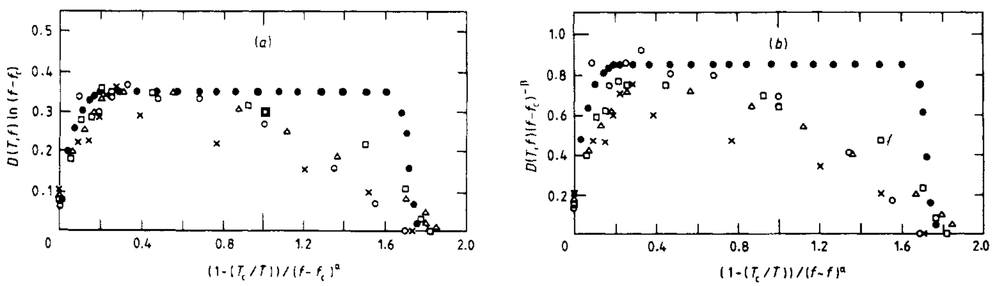

Figure 2. Average scaled damage $D(T,f)$ against $(1 - T_c/T)/(f - f_c)^\alpha$ ($\alpha = 0.83$) using for the vertical axis the scaling forms (*a*) $D(T,f)|\ln(f - f_c)|^{-1}$, (*b*) $D(T,f)(f - f_c)^{-\beta}$ ($\beta = 0.76$). The symbols have the same meaning as in figure 1.

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